Computer Method for Perspective Drawing

H. R. Puckett*
Douglas Aircraft Company, Inc., Santa Monica, Calif.

A computer with a line plotter can produce perspective drawings useful to depict proposed structures prior to stress analysis, compare deflected structures with their unloaded configurations, show pressure or stress variations over surfaces, and improve spatial visualization. The computer converts rectangular coordinates of "defining points" into those necessary for a perspective view by rotating axes through arbitrary tilt and turn angles, translating axes to "center" the origin, and computing the shifting necessary for perspective projection. Developed point numbers are joined, as appropriate, by line segments. Stereographic pairs may be produced as an option; offsets for left and right eye viewing are determined during the computations for the central projection.

Introduction

THROUGH the ages man has attempted to record three-dimensional objects on two-dimensional surfaces. Techniques (freehand art, drafting, and photography) have been developed for reducing solids to planes without loss of identity. Draftsmen, in addition to transmitting technical information by use of conventional principal-axes drawings, have improved communication through the use of pictorials.[1] Widely used are isometric, oblique, and trimetric views; true perspective is avoided because of the difficulty of rendition. This paper presents a less tedious perspective drawing method using an electronic computer equipped with a line plotter.

Computers may be analog or digital. Analog devices such as slide rules, thermometers, and most clocks produce a continuous description of the variable; interpolation is automatic. Digital devices, like the abacus, typical adding machines and desk calculators, and our money system express quantity in finite increments. Sensitivity can be improved by taking smaller increments, but the increments are always finite. Drafting is essentially analog, but the basic nonperspective information may be processed by digital means and converted, at a late stage, to an analog representation, that is, into a perspective drawing.

Points, lines, planes, and solids, the basic components of geometry, may all in the ultimate be defined by points alone, since the locus of a point defines a line, the locus of a line defines a plane, and the locus of a plane defines a solid. The draftsman has chosen the line as his basic element; his pictures are an integration of many lines. The computer method given here follows this approach in its last step only. Before that time, the computer deals only with points; those points which lie at the ends of the line segments to be depicted. Curves may be approximated by many short straight line segments.

Method

From the foregoing, one sees that the input to the computer must contain the coordinates of all the defining points of the object. Also, in order that the sight direction may be optional, input must contain angles of turn, tilt, and in some cases, aspect. The first phase of the computer operation converts these data into a new set of coordinates for the object rotated into the proper viewing position.

Presented as Preprint 2920-63 at the AIAA Launch and Space Vehicle Shell Structures Conference, Palm Springs, Calif., April 1–3, 1963; revision received September 30, 1963.

* Structures Engineer, Missile and Space Systems Division.

Rotation

The mathematics for this rotation comes from elementary analytical geometry. The equations are as follows:

$$X'' = X''' \cos\theta \cos\beta - Y''' \sin\theta \cos\beta + Z''' \sin\beta$$

$$Y'' = X''' (\sin\theta \cos\gamma + \cos\theta \sin\beta \sin\gamma) + Y''' (\cos\theta \cos\gamma - \sin\theta \sin\beta \sin\gamma) - Z''' \cos\beta \sin\gamma$$

$$Z'' = X''' (\sin\theta \sin\gamma - \cos\theta \sin\beta \cos\gamma) + Y''' (\cos\theta \sin\gamma + \sin\theta \sin\beta \cos\gamma) + Z''' \cos\beta \cos\gamma$$

where θ is the angle of turn, β is the angle of tilt, γ is the aspect angle; X''', Y''', and Z''' are the coordinate values of a given point in the original orientation.

The equations are based on an arrangement in which the original sight direction is along the X axis from plus toward minus; the plus Y is to the right and the plus Z is up. The angle of turn θ is made first and is a rotation about the Z axis. θ is taken positive when the turn is counterclockwise as viewed from plus Z toward minus Z. The rotation about the Y axis is next, the counterclockwise angle of tilt β being considered positive when viewed from plus Y toward minus Y. The aspect angle γ is usually not needed. If present, it is about the X axis and is positive when counterclockwise as viewed from plus X toward minus X. Other arrangements could have been established provided the equations were revised accordingly.

The computer applies the foregoing equations to every point of the data. In addition, it compares the new values to determine the maximum X'', Y'', and Z'' and the minimum Y'' and Z''. These extremes are needed in later phases.

Translation

The perspective projection which is to follow will be more easily computed if a picture plane is established passing through the foremost point of the object and perpendicular to the sight line. Also, the object should be shifted so that the sight line passes through the center of the object. It is desirable to reverse the signs for the X values. These requirements can be met by the use of the equations of translation which yield a third set of coordinate values of the defining points:

$$\begin{aligned} p &= (X''_{\max}) \\ q &= 0.5(Y''_{\max} + Y''_{\min}) \\ r &= 0.5(Z''_{\max} + Z''_{\min}) \\ X' &= -(X'' - p) \\ Y' &= Y'' - q \\ Z' &= Z'' - r \end{aligned}$$

where X', Y', and Z' are the coordinates of a given point after rotation and translation.

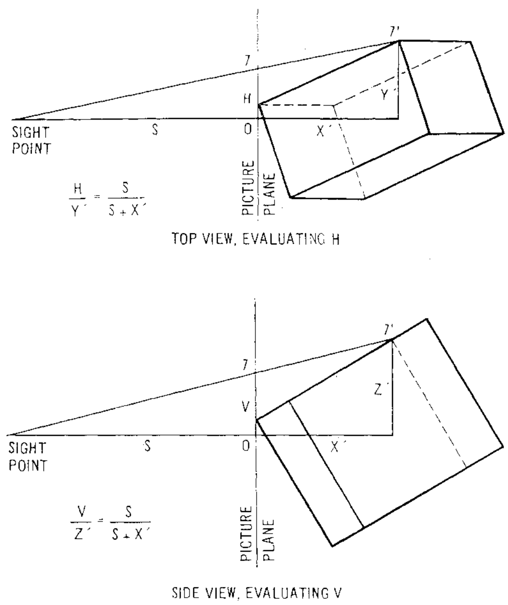


Fig. 1 Perspective theory.

Perspective Projection

Perspective projection, whether by manual means or computed, consists of finding the points of intersection of the picture plane with lines extending from defining points on the object to the sight point (eye position). Figure 1 shows the geometry involved. The equations based on this figure are

$$
\begin{aligned}
d &= [(Y''_{\max} - Y''_{\min})^2 + (Z''_{\max} - Z''_{\min})^2]^{1/2} \\
s &= kd \\
t &= s/(s + X') \\
H &= Y't \\
V &= Z't
\end{aligned}
$$

where d is the diagonal of the rotated view, k the sight distance ratio (input data), s the distance from sight point to picture plane, and H and V are the horizontal and vertical coordinate values for a given point in the final perspective drawing. The evaluation of H and V is, of course, accomplished by the computer for each defining point.

Plotting

Plotters are available which will connect points with straight line segments, or which can be made to move from point to point without producing a line. When connected to an electronic computer, the plotter operations can be program controlled so that the total process is automatic. Additional input to the computer is necessary at this point to indicate the sequence in which line segments are to be drawn or skipped and to scale the drawing for the particular view size desired. Pen lift (no line) can be indicated by the presence of zeros in the proper locations in the sequence of point numbers. The computer tests for these zeros and instructs the plotter accordingly.

The matter of scaling can be handled nicely by inputting to the computer the width and height of the paper area to be provided. The scale is then automatically computed from the following equations. It should be noted that the scales for H and V must be equal and that the one that allows the filling of the allotted space is caused to prevail:

$$SF_H = (Y''_{\max} - Y''_{\min})/L_H$$

$$SF_V = (Z''_{\max} - Z''_{\min})/L_V$$

$$SF = 0.5(SF_H + SF_V + |SF_H - SF_V|)$$

where L_H is the width of paper area into which view must fit, L_V the height of paper area into which view must fit, SF_H the scale factor based on width, SF_V the scale factor based on height, and SF the larger of SF_H and SF_V.

Uses

The greatest advantage of the computer method over other methods for producing perspective drawings is evidenced when the original data is in numerical form. This is often the case when the item to be depicted is in the idea stage and the geometry has been computed. The method was originally devised to provide drawings necessary to show idealized structures for stress analyses at the Douglas Aircraft Company. For this use, the computer produces a drawing as shown in Fig. 2. Using this, the engineer can produce drawings for his purposes as shown in Fig. 3.

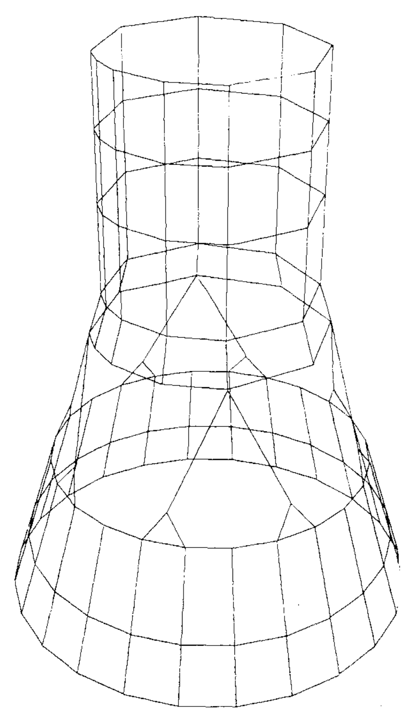
Graphs between three variables have the data in numerical form and lend themselves to pictorial display. One variation of this usage is shown in Fig. 4, where two of the axes represent the two dimensions of a flat representation of a structure undergoing stress. The third axis shows the stress magnitude at the various points of the structure. Two different stress conditions are shown. Using a polar coordinate representation, Fig. 5 shows the distribution of aerodynamic pressure over the surface of a missile.

Other possible uses are in the fields of architecture, interior decoration, machine design, etc. For applications for which the computer does not provide the proper finish, it can nevertheless yield an accurate skeleton view upon which hand work can be added, thus reducing the tedious and time-consuming work for the draftsman or artist.

Computers

Any digital electronic computer that is capable of driving a line plotter can be programmed to create perspective pictures in 2-D or 3-D. The equipment used to produce most of the pictures shown here was a Bendix G-15D computer and a Cal-Comp plotter. Figure 4, including lettering, was drawn by a Cal-Comp plotter driven by a magnetic tape unit. The tape was produced from a Fortran program on an IBM 7094 computer.

Fig. 2 Delta interstage idealized for stress analysis. Computer drawn.



Variations

Most objects are either rectilinear or may be readily referred to three mutually perpendicular rectilinear axes. These are the principal axes. If θ and β are chosen such that two of the principal axes are parallel to the picture plane (perpendicular to the sight line), the drawing will be "one-point perspective." If only one principal axis is parallel to the picture plane, the drawing will be a "two-point perspective." If none are parallel, a "three-point perspective" will result. Since most objects are originally oriented with two principal axes parallel to the picture plane, the forementioned situations would result when θ and β were, respectively, both zero, only one zero, neither zero.

Drawing an analogy from photography, the assigning of unity to the factor k is like using a normal lens on a camera. A value of 0.7 would produce a view similar to that taken with a "wide angle" lens, whereas values of 1.5, 3, 5, etc., would be like using telephoto lenses of various magnifications. It should be realized that the perspective effect is lost in telephoto pictures. Similarly, if we make k large in the computer solution, the view approaches an orthographic view. A large k was used for Fig. 4.

The plotter used by the author may be equipped with either ball-point or India-ink pens. This allows the use of a variety of colors. Contrast between parts of a drawing can, therefore, be effected by using different colors.

Stereoscopy

The human eyes, being separated by several inches, observe slightly different views of objects being seen. This enables the observer to compute subconsciously by triangulation the distance to various points within the view. This phenomenon can be maintained in graphical representation if perspective pictures are produced from two different sight points corresponding to the two eye positions and later presented to the observer in such a manner that the picture produced with the left-eye sight point is seen only by the ob-

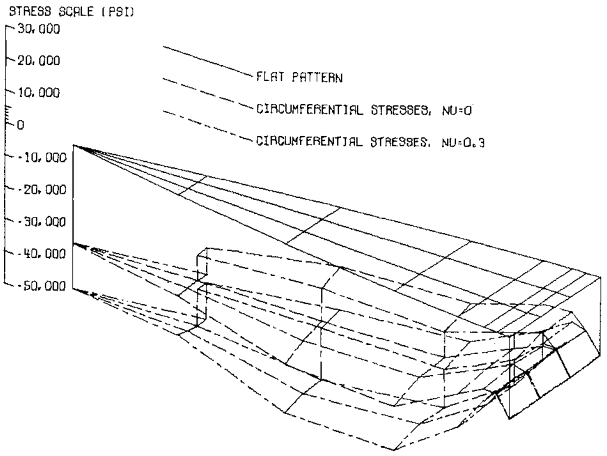


Fig. 4 Forward face common bulkhead circumferential stresses temperature condition.

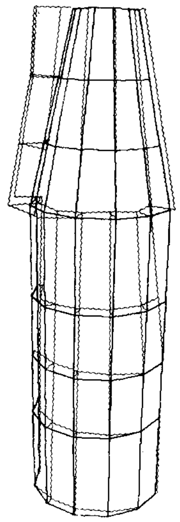


Fig. 5 Low drag pressure distribution, Delta vehicle.

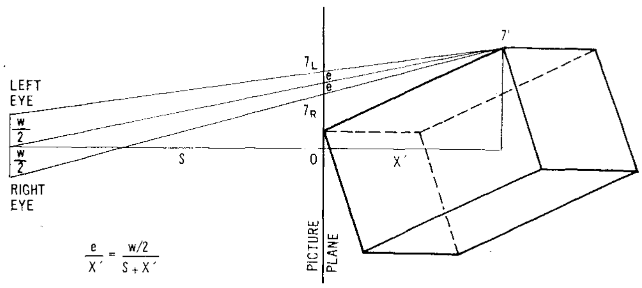


Fig. 6 Perspective theory. Top view, evaluating *e*.

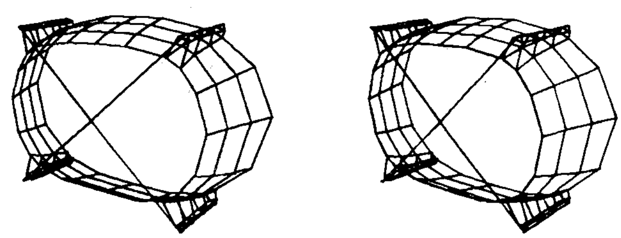


Fig. 7 Nose cabin of Astro vehicle.

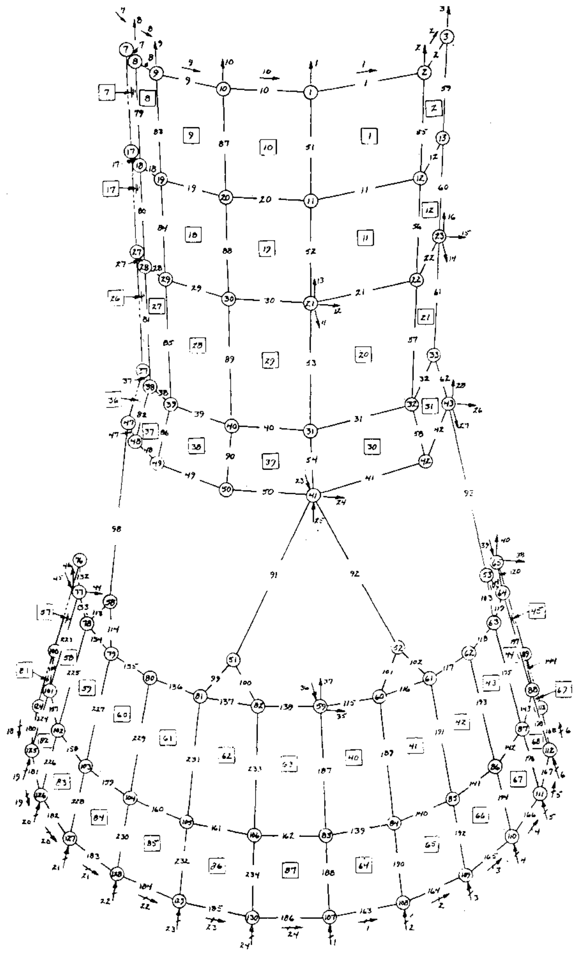


Fig. 3 Manual additions to Fig. 2 for needs of engineers.

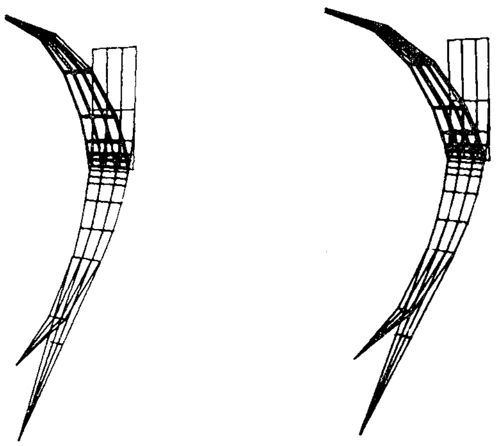


Fig. 8 Section through Saturn vehicle showing spherical bulkhead.

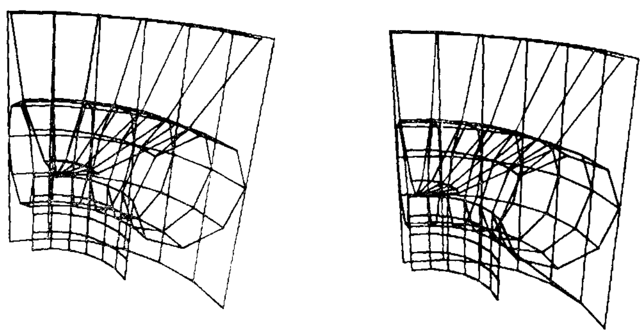


Fig. 9 Section through space vehicle in region of tanks.

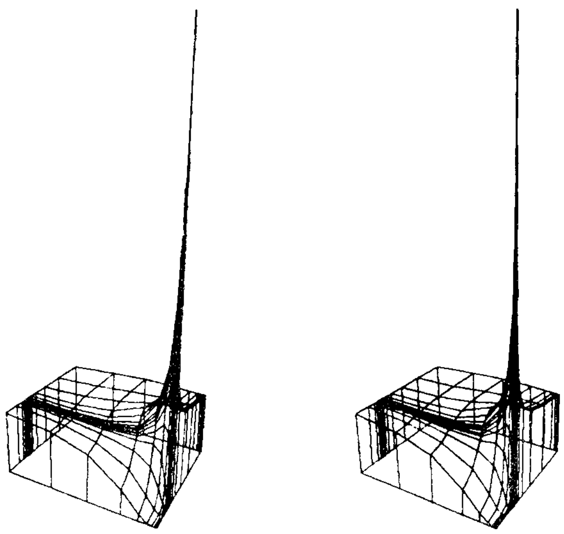


Fig. 10 Stress distribution in cracked plate; only one quarter shown.

server's left eye, and the one produced using the right-eye sight point is observed only by the observer's right eye. Familiar examples of devices to accomplish this are the old-fashioned parlor stereoscope with cards carrying photographic stereo pairs, and the modern cameras and viewers for making and observing 35-mm color transparencies in 3-D.

Since the production of these stereo pairs is only a matter of creating two pictures from slightly different sight points, the computer method for 2-D pictures set forth in the foregoing needs only to be modified according to the theory shown in Fig. 6. The equations are as follows:

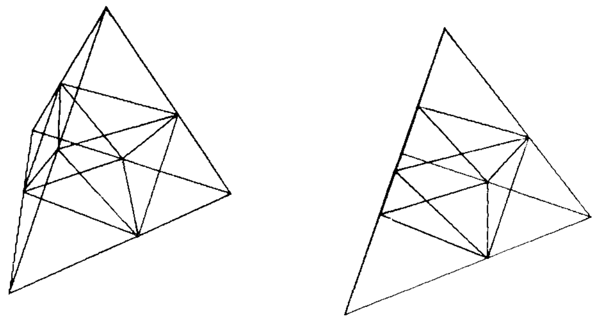
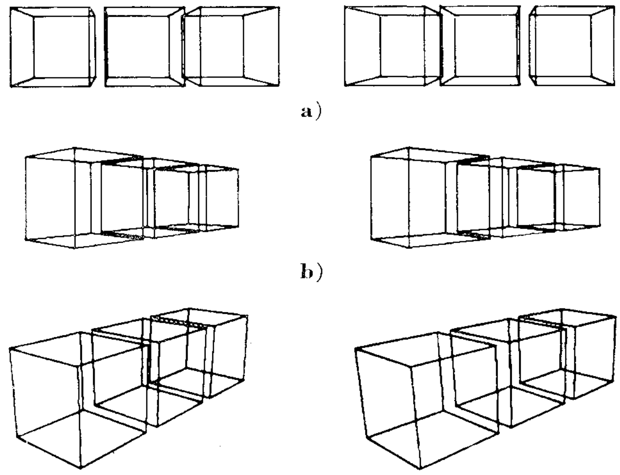


Fig. 11 Geometric figure to show how steroscopy gives meaning to unfamiliar object.



a)

b)

c)

Fig. 12 One-, two-, and three-point perspective (a, b, and c, respectively).

$$v = X'/(s + X')$$

$$e = (wv)/2$$

$$H_L = H - e$$

$$H_R = H + e$$

where e is the offset for a given point for stereo pairs, w the eye-to-eye distance (input data), and H_L and H_R the horizontal coordinate value for a given point for the left and right stereo views, respectively. The computer evaluates e, H_L, and H_R for each point used to describe the object. The values of V to be used are the same as those for a 2-D picture, and the values of H differ only by the computed offset e.

Stereoscopy adds interest and definition to any picture but finds it greatest usefulness in situations in which the observer is not familiar with the object pictured. Figures 7–12 show examples of stereo pairs produced by the computer.

Stereoscopy by computer could be used in the study of space lattice arrangements of molecules in crystals, for trajectories in space, to show curved surfaces in space, and wherever depth relationships are important. In some instances drawings which are complicated by many lines are clear in 3-D whereas they would not be in a 2-D picture. See, for instance, Fig. 9.

Optimum viewing of 3-D pictures requires devices to aid in the separate presentation of the views to the two eyes. These viewers take various forms.[2] It is possible, with practice and perseverance, and if the views are properly sized and spaced, to observe stereo pairs without optical aids. The examples shown herewith are spaced approximately 1.75 in. apart, which is close enough to allow observation by persons with the smallest eye-to-eye measurement. Observing without a stereoscope yields three views, the middle one being in 3-D. This middle view can be captured by first looking past the paper at some object about three times as far from the eye as the views are, seeing the three views out of the bottom of the eyes, and then finally focusing on the middle view. It takes an appreciable time for the final focus to become effective. Horizontal alignment is essential.

References

[1] French, T. E. and Vierck, C. J., *A Manual of Engineering Drawing* (McGraw-Hill Book Company, Inc., New York, 1953), 8th ed., Chaps. 10, 11, 16, and 17.

[2] Martin, L. C., *Technical Optics* (Sir Issac Pitman and Sons, London, 1954), Vol. 2.

JANUARY 1964 J. SPACECRAFT VOL. 1, NO. 1

Impact of Dislocation Theory on Engineering

John E. Dorn* and Jack B. Mitchell†
University of California, Berkeley, Calif.

Dislocation theory is discussed in terms of the nature and properties of individual dislocations and their interactions. Examples are given illustrating how dislocation theory permits a description of the mechanical behavior of crystalline materials in terms of mathematical, physical, and chemical concepts familiar to engineers. Although the paper presents several direct results of the application of dislocation theory to the development of engineering materials, the major emphasis is given to the value of dislocation theory in the development of a sound philosophical background and simple analytical procedures for the development of new crystalline materials and for predicting how real materials might behave under new and different environmental conditions.

Introduction

HINTS concerning the possible importance of some dislocation-like imperfections in crystalline materials were announced in 1928 and 1929 by Prandtl[1] and Dehlinger.[2] In 1934 Taylor,[3] Orowan,[4] and Polanyi[5] independently "invented" models of slip dislocations on which the presently accepted dislocation theory is based. Progress in extending this new engineering science was extremely slow prior to 1948. This delay might, in part, be attributed to interruptions by World War II, but it was principally because 1) no one had truely "seen" dislocations except in terms of very indirect evidence, and 2) several different investigators could marshall evidence to "describe" the same experimental facts in terms of several unique and often apparently contradictory dislocation models. It is now recognized that dislocations are extremely versatile, a fact that stimulates the imagination and warns against hasty and naïve interpretations.

Since 1948 great progress has been made in our grasp of the subject of dislocations: 1) dislocations can now be "seen" in a number of ways: 2) a vast body of sound theory has been generated; and 3) a host of experimental observations can now be uniquely and accurately analyzed in terms of the theory. On the other hand, much yet needs to be done before a full and accurate account of all issues can be presented, and because dislocations are so versatile, theory and experiment must yet be advanced simultaneously, each complementing the other in arriving at the truth.

Dislocation theory has been, and is currently being, applied toward the development of new alloys. It has led to the development of nonstrain-aging automotive sheet and to the development of extremely high-strength ausformed steels, and it is now being applied to the development of new creep-resistant alloys of the refractory metals and to the possible production of ductile ceramics. Such direct examples of applications are easily appreciated. However, there is another perhaps greater, but less obvious, significance of dislocation theory to the engineer: It gives him a sound analytical basis in terms of atomistics and the mathematical theory of elasticity on which to base his judgment not only regarding the development of new crystalline materials having special desirable properties but also with respect to predictions of how real materials might behave under new and different environmental conditions. It will help him to decide whether or not the plastic behavior of a new material will be temperature- or strain-rate sensitive and whether the flow stress will be so high at low temperatures that notch-sensitivity and brittleness may be imminent, and it will provide the basis for formulating the constitutive equations essential to the solution of problems of deformation of materials.

Since this paper is being written primarily for those engineers who have only a modest background in the subject, some of the elementary concepts on the nature of dislocations, their stress fields, and energies will be reviewed before other more sophisticated problems are illustrated.

Nature of Dislocations

As shown in Fig. 1, the theoretical resolved shear stress τ for slip in an ideal single crystal is 10% of the shear modulus of elasticity. Extremely small whiskers of metallic and

---

Presented as Preprint 63-236 at the AIAA Summer Meeting, Los Angeles, Calif., June 17-20, 1963; revision received October 21, 1963. The authors express their appreciation to the U. S. Atomic Energy Commission for their support of this research conducted under the Inorganic Materials Research Division of the Lawrence Radiation Laboratory in Berkeley. They further wish to thank Jim Mote for his advice and help in preparing this paper.

* Miller Professor of Materials Science 1962-63, Department of Mineral Technology; also Research Metallurgist, Inorganic Materials Research Division, Lawrence Radiation Laboratory.

† Research Metallurgist, Inorganic Materials Research Division, Lawrence Radiation Laboratory.